mentioned earlier, this chattering arises from the excessive supply of control input to relatively small oscillations. This chattering is reduced for the lower levels of control current by trading off decay time.

By employing the PAC, the chattering was completely eliminated without deteriorating the decay time. This implies that the PAC algorithm produces relatively small adverse control force associated with the input current in the settled phase. The threshold tip deflection (y_{e1}), which decides the activation time of the PAC, was 0.73 mm in this realization. On the other hand, for the implementation of the SMC the discontinuous feedback gain $k = 0.7$ was employed. With the feedback gain and imposed initial conditions, the optimal value of $c = 6.9$ was determined to minimize the performance index J given by Eq. (10). It is seen that the tip deflection was favorably suppressed without the unbalancing phenomenon that occurred in the CAC and the PAC. Note that the control characteristics of the SMC depend on the design parameters k and c.

Conclusion

An active vibration control utilizing the SMA actuators has been undertaken to suppress the bending vibrations of a cantilevered beam. A modified CAC was proposed and successfully implemented to eliminate undesirable chattering in the settled phase. In addition, superior control performance was demonstrated by employing the SMC. The control effects attributable to the number of actuators and the location of the actuators are to be studied in the future.

References

[1]Baz, A., Imam, K., and McCoy, J., "Active Vibration Control of Flexible Beams Using Memory Actuators," *Journal of Sound and Vibration*, Vol. 140, No. 3, 1990, pp. 437–456.

[2]Choi, S. B., and Park, D. W., "Moving Sliding Surfaces for Fast Tracking Control of Second-Order Dynamical Systems," *Journal of Dynamic Systems, Measurement and Control*, Vol. 116, No. 1, 1994, pp. 154–158.

Near-Optimal Operation of Dual-Fuel Launch Vehicles

M. D. Ardema* and H.-C. Chou†
Santa Clara University, Santa Clara, California 95053
and
J. V. Bowles‡
NASA Ames Research Center, Moffett Field, California 94035

Introduction

CURRENT studies of single-stage-to-orbit (SSTO) launch vehicles are focused on all-rocket propulsion systems.[1,2] One option for such vehicles is the use of dual fuel [liquid hydrocarbon and liquid hydrogen (LH_2)] for a portion of the mission.[3–6] Compared with LH_2, hydrocarbon fuel has higher density and produces higher thrust to weight but has lower specific impulse. These advantages of hydrocarbon fuel are important early in the ascent trajectory, where vehicle weight is high, and its use may be expected to lead to reduced vehicle size and weight. Because LH_2 is also typically needed for cooling purposes, in the early portion of the trajectory both fuels usually must be burned simultaneously. Later in the ascent, when vehicle weight is lower and thrust requirements are less, specific impulse is the key parameter, indicating single-fuel LH_2 use.

Two recent papers[5,6] have considered the optimization of dual-fuel SSTO vehicles. Included in the studies was a determination of M^*_{tr}, the Mach number at which to transition from dual-fuel mode to LH_2 operation to minimize vehicle empty weight. Both of these references treat M_{tr} as an external design parameter, which must be optimized by iterative design evaluations.

In this Note, a guidance algorithm is developed that determines whether dual-fuel or single-fuel operation is superior as an integral part of the trajectory integration. This approach saves a substantial number of iterations of a computer design code by reducing the number of design variables and hence the number of design iterations required in a vehicle optimization study. Further, the guidance law will be directly usable as part of a real-time, onboard propulsion control system.

The basis of the guidance law is the energy-state dynamic model. The key idea is to introduce the total mechanical energy as a state variable and then to neglect all other dynamics. This results in a function optimization problem. When flight-path optimization is done with this model, simple rules for the optimal path and for the optimal operation of propulsion system are obtained. This dynamic model has been used successfully many times to obtain effective guidance laws for a wide variety of aircraft and missions (see Ref. 7 and the references therein for a review of this work). The energy-state approach is particularly suitable for launch vehicles because efficient energy accumulation (or equivalently maximizing total ΔV) is the primary trajectory optimization goal.

In a series of papers,[7–9] we have used energy-state methods to develop algorithms for ascent trajectory optimization and optimal operation of single-fuel multimode propulsion systems. In particular, the operation of propulsion systems with two separate engines, airbreathing and rocket, was investigated.[9] The present Note extends those methods to the dual-fuel case. The main goal is to determine M^*_{tr} and to investigate optimal trajectories.

In the numerical results, vehicle performance is computed using the NASA Ames Research Center code HAVOC. HAVOC integrates geometry, aerodynamics, propulsion, structures, weights, and other computations to produce point designs for a wide variety of launch vehicles. It is capable of iteratively determining closed vehicles, that is, designs that meet specified payload mass and volume requirements for a specified mission. Although the trajectory guidance law is based on the energy-state model, the trajectory integration in HAVOC uses a point-mass model, including the effects of the Earth's rotation, the Earth's curvature, and variable gravity.

Optimization Function

The energy-state model is obtained by using the total mechanical energy per unit weight as the state variable[7–9]:

$$\dot{E} = P \tag{1}$$

$$\dot{W} = -(T/I_{SP}) \tag{2}$$

where

$$E = [hR/(R+h)] + (1/2g)V^2 \tag{3}$$

and

$$P = (V/W)(T_v - D) \tag{4}$$

and where the drag is evaluated at the lift required for equilibrium of forces perpendicular to the flight path. In these equations, E, P, W, T, I_{SP}, h, R, V, g, T_v, and D are energy per unit weight, specific excess power, weight, thrust, specific impulse, altitude, the Earth's radius, gravitational acceleration at the Earth's surface, component of thrust along the velocity vector, and drag, respectively.

For an SSTO mission, what is desired is a trajectory that gives the minimum-empty-weight vehicle to put a given payload mass and volume in orbit. Because the density of liquid hydrogen is low, the sensitivity of perturbations in volume needs to be taken into consideration as well as mass sensitivity, and it is therefore necessary

Received Jan. 29, 1996; revision received May 9, 1996; accepted for publication June 13, 1996.

*Professor and Chairman, Department of Mechanical Engineering. Associate Fellow AIAA.

†Graduate Research Assistant, Department of Mechanical Engineering. Student Member AIAA.

‡Aerospace Engineer, System Analysis Branch. Member AIAA.

to minimize a weighted sum of propellant weight and volume. Thus we introduce the cost functional

$$\phi = W_P + K v_P \tag{5}$$

where $K \in [0, \infty)$ is a weighting parameter to be chosen later.

The quantity to be minimized for a given energy gain is

$$J' = \int_{\phi_0}^{\phi_f} \mathrm{d}\phi = \int_{t_0}^{t_f} \dot{\phi}\,\mathrm{d}t = \int_{E_0}^{E_f} \frac{\dot{\phi}}{P}\,\mathrm{d}E \tag{6}$$

where Eq. (1) was used. It is assumed that $\dot{\phi} > 0$, $P > 0$, and E is monotonically increasing. If the propellant density is $\rho = W_P/v_P$, then from Eqs. (2) and (5), and using $\dot{W}_P = -\dot{W}$,

$$\dot{\phi} = \dot{W}_P + K(\dot{W}_P/\rho) = (T/I_{\mathrm{SP}})[1 + (K/\rho)] \tag{7}$$

For convenience, we choose to invert the integrand in Eq. (6) and maximize; from Eqs. (1), (6), and (7), the quantity to be maximized is

$$J = \int_{E_0}^{E_f} F\,\mathrm{d}E \tag{8}$$

where

$$F = \frac{V I_{\mathrm{SP}}(T_v - D)}{W T[1 + (K/\rho)]} \tag{9}$$

The guidance algorithm then consists of selecting propulsion system and trajectory parameters that maximize the function F as given by Eq. (9) at each energy level along the trajectory, subject to any relevant constraints.

For vehicles capable of either dual- or single-fuel operation, the densities to be used in Eq. (9) are

$$\rho_{\mathrm{DF}} = \frac{\rho_R \rho_O \rho_H (1 + \eta_{OR} + \eta_{HR})}{(\rho_O \rho_H + \eta_{OR} \rho_R \rho_H + \eta_{HR} \rho_R \rho_O)} \tag{10}$$

$$\rho_{\mathrm{SF}} = \frac{\rho_O \rho_H (1 + \eta_{OH})}{(\rho_O + \eta_{OH} \rho_H)} \tag{11}$$

where subscripts DF, SF, R, O, and H refer to dual fuel, single fuel, hydrocarbon, oxygen, and hydrogen and η_{HR}, η_{OH}, and η_{OR} are mass-flow ratios.

SSTO vehicles are typically subject to dynamic pressure constraints and a maximum tangential acceleration limit. The latter limit, nominally 3 g, is met by engine throttling. It may happen that the limit affects dual-fuel operation but not single-fuel operation at a point along the trajectory. All of these constraints are accounted for in the guidance algorithm.

In addition to being a useful tool in preliminary design studies, the guidance algorithm should be ideal for use in an onboard real-time control system because 1) it is fully nonlinear and models all of the vehicle's significant nonlinearities, 2) it is algebraic and thus does not rely on potentially unstable numerical integrations, and 3) it depends directly on easily measured vehicle states and parameters.

Numerical Results

All numerical examples will be based on an SSTO rocket with a delta winged-body configuration.[2] The three propellants (hydrocarbon fuel, LH_2, and liquid oxygen) are stored in three separate internal tanks. The vehicle takes off vertically and lands horizontally. The first results to be presented use a fixed trajectory commonly used for SSTO rockets.

As a first step, the best transition Mach number M_{tr}^* was determined by treating this parameter as a single external design variable, as was done in Refs. 5 and 6. This required several iterations of the HAVOC. The results are that both minimum liftoff weight (W_{LO}) and minimum empty weight (W_E) are obtained at about $M_{\mathrm{tr}}^* = 9.0$ and that the weight savings at M_{tr}^* are substantial relative to low values of M_{tr}.

Before applying the developed guidance law to this problem, the best value of K must be determined. This is done by computing closed vehicles for a range of values of K. It was found that a value of $K = 4$ lb/ft^3, denoted hereafter by K^*, gives very nearly a minimum of both empty weight and gross liftoff weight, and this value will be used throughout the rest of the paper. This value of K^* represents a factor of over 10 in weighting the cost functional in favor of propellant mass (a value of $K = \rho$ would signify equal weighting of propellant mass and volume). The use of the optimally weighted cost functional saves 1.7% in empty weight and 1% in gross liftoff weight, relative to minimizing propellant weight only.

It is of interest to compare these results with the equivalent results for an air-breathing launch vehicle, as shown in Fig. 2 of Ref. 9. For the air breather, the best value of K is also around 4, but the empty weight reduction relative to minimizing propellant weight only is much larger, at 4.9%; this is, of course, because all of the air-breather propellant is low-density LH_2, and therefore this vehicle is more sensitive to volume perturbations.

Figure 1 plots the function F along the fixed trajectory. Whichever mode of operation, dual-fuel or single-fuel, that gives the highest value of F at a given speed should be the one selected at that speed. The figure shows that from liftoff to $M = 9.0$ the dual-fuel mode is superior, and above this speed the single-fuel mode is best. This value of $M_{\mathrm{tr}}^* = 9.0$ agrees with the value determined by treating M_{tr} as a design variable, thus validating the guidance law. The value of M_{tr}^* as determined in Ref. 5 was in the range 8.6–8.9, and for Ref. 6 it was in the range 7.3–7.4.

The relative distance between the two curves on Fig. 1 provides an assessment of the difference in performance between the two modes at a given Mach number. It is seen that both modes give substantially the same performance between $M = 7$ and 11. This relative insensitivity to M_{tr}, characteristic of a design variable near its optimal value, was also observed in Ref. 5. The use of the single-fuel LH_2 mode becomes increasingly advantageous as Mach number increases past 11.

The function F was also used to optimize the ascent trajectory (Fig. 2). The dashed lines running between the maximum and minimum dynamic pressure limits are lines of constant energy. Compared with the fixed trajectory, the near-optimal one has increased dynamic pressure, especially in the initial dual-fuel mode. The plot of F for the two modes along the optimal trajectory is very similar to Fig. 1, except that M_{tr}^* is now 9.6. The near-optimal trajectory consumed less fuel in the amount of 0.9% of W_{LO} than did the fixed, almost all of the difference occurring in dual-fuel mode.

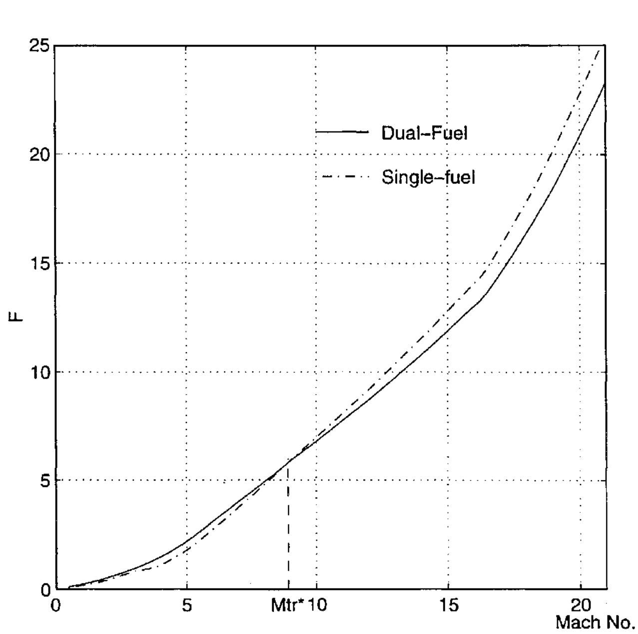

Fig. 1 Cost functional histories for dual- and single-fuel propulsion modes.

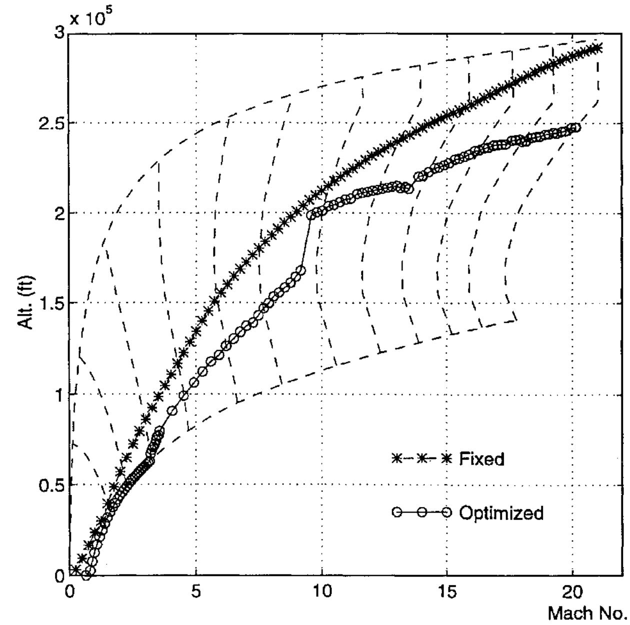

Fig. 2 Fixed and optimized flight paths.

Conclusion

A simple guidance law for operation of dual-fuel SSTO launch vehicles has been developed and used to determine the optimal value of the transition Mach number from dual fuel to single fuel. For the example considered, the optimal transition Mach number was 9.0 along a fixed trajectory. Along an optimal trajectory, the best transition Mach number was 9.6; the optimal trajectory had higher dynamic pressure than the fixed, particularly in dual-fuel mode.

In the future, the guidance method described in this Note easily could be extended to optimize other propulsion system parameters, such as flow rates of individual propellants in multipropellant engines. Because the guidance algorithm is internal to the trajectory optimization routine, its use will save many iterations of a preliminary design computer code relative to treating these parameters as external design variables. The guidance law is highly accurate, robust, and simple to implement, also making it ideal for use in a real-time onboard control system for an SSTO launch vehicles.

Acknowledgment

This work was supported by NASA Ames Research Center Grant NCC 2-5069.

References

[1]Bekey, I., "SSTO Rockets; A Practical Possibility," *Aerospace America*, Vol. 32, No. 7, 1994, pp. 32–37.

[2]Freeman, D. C., Talay, T. A., Stanley, D. O., Lepsch, R. A., and Wilhite, A. W., "Design Options for Advanced Manned Launch Systems," *Journal of Spacecraft and Rockets*, Vol. 32, No. 2, 1995, pp. 241–249.

[3]Salkeld, R., "Mixed-Mode Propulsion for the Space Shuttle," *Astronautics and Aeronautics*, Vol. 9, No. 8, 1971, pp. 52–58.

[4]Wilhite, A. W., "Optimization of Rocket Propulsion Systems for Advanced Earth-to-Orbit Shuttles," *Journal of Spacecraft and Rockets*, Vol. 17, No. 2, 1980, pp. 99–104.

[5]Lepsch, R. A., Stanley, D. O., and Unal, R., "Dual-Fuel Propulsion in Single-State Advanced Manned Launch System Vehicle," *Journal of Spacecraft and Rockets*, Vol. 32, No. 3, 1995, pp. 417–425.

[6]Braun, R. D., Powel, R. W., Lepsch, R. A., Stanley, D. O., and Kroo, I. M., "Comparison of Two Multidisciplinary Optimization Strategies for Launch-Vehicle Design," *Journal of Spacecraft and Rockets*, Vol. 32, No. 3, 1995, pp. 404–410.

[7]Ardema, M. D., Bowles, J. V., and Whittaker, T., "Optimal Trajectories for Hypersonic Launch Vehicles," *Dynamics and Control*, Vol. 4, No. 4, 1994, pp. 337–347.

[8]Ardema, M. D., Bowles, J. V., Terjesen, E. J., and Whittaker, T., "Approximate Altitude Transitions for High-Speed Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 3, 1995, pp. 561–566.

[9]Ardema, M. D., Bowles, J. V., and Whittaker, T., "Near-Optimal Propulsion-System Operation for an Air-Breathing Launch Vehicle," *Journal of Spacecraft and Rockets*, Vol. 32, No. 6, 1995, pp. 951–956.

Effect of Damping on the Structure of the Time-Optimal Control Profile

T. Singh*
State University of New York at Buffalo, Buffalo, New York 14260

I. Introduction

THERE exists a significant body of work that addresses the design of time-optimal controllers. Singh et al.,[1] Ben-Asher et al.[2] Liu and Wie,[3] and Singh and Vadali[4] study the problem of design of time-optimal controllers for flexible slewing structures, which are represented by finite-dimensional linear dynamical models. Liu and Wie[3] and Singh and Vadali[4] also address the problem of desensitizing the time-optimal controller to errors in system parameters. The problem of time-optimal reorientation of spacecrafts has been addressed by Billimoria and Wie,[5] who noted a change in the control structure from a seven-switch to a five-switch profile with an increase in the maneuver. A survey of the problem of time-optimal attitude maneuvers has been provided by Scrivener and Thompson.[6] Pao,[7] via a parametric study of a damped floating oscillator, exemplified the existence of three- and five-switch time-optimal control profiles. The damping ratios corresponding to the transition of the control structure were determined by a parametric study.

This Note proposes a simple technique to determine the transition of the control profile from one structure to another. A damped floating oscillator with the same system parameters as used by Pao[7] is considered to illustrate the proposed technique, where the values of the damping ratio that corresponds to the transition from a three switch to a five switch and, finally, to a three-switch control profile are determined.

II. Existence of Control Profile Transitions

The time optimal control for the system

$$\dot{x} = Ax + bu \tag{1}$$

is given by the equation

$$u = -\mathrm{sgn}(\lambda^T b) \tag{2}$$

where $x \in R^n$ is a vector of system states, $\lambda \in R^n$ is the costates vector, and $u \in R^m$ is the control vector.

It can be seen from Eq. (2) that the control input changes sign when the switching function $\lambda^T b$ passes through zero. With the variation of certain parameters in the control problem, additional switches can be introduced into the control structure either at the initial time or final time or at some time in between the initial and final time. If there exists a control profile transition effected by switches appearing at the initial or final time of the ith control input, the equation that has to be satisfied is

$$(\lambda^T b)_i = 0 \tag{3}$$

at the initial or final time, respectively, since only one switch can enter from either end of the time boundaries. The variation with respect to the system parameters of the initial costates has been assumed to be smooth. If two switches are introduced simultaneously at both ends of the time boundaries, then Eq. (3) has to be simultaneously satisfied at the corresponding time instants. Since the costates are smooth functions of time, if the switches appear in between the initial and final time, then two switches are introduced simultaneously and the transition occurs when the equations

$$(\lambda^T b)_i = 0 \tag{4}$$

Received May 31, 1995; revision received Aug. 8, 1995; accepted for publication March 27, 1996.

*Assistant Professor, Department of Mechanical and Aerospace Engineering. Member AIAA.